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Induced gravity on RS branes

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Abstract: It is shown that a localized four-dimensional Einstein term, induced by quantum corrections, modifies significantly the law of gravity in a Randall-Sundrum brane world. In particular, the short-distance behavior of gravity changes from five- to four-dimensional, while, depending on the values of parameters, there can be an intermediate range where gravity behaves as in five dimensions. The spectrum of graviton fluctuations around the brane, their relative importance for the gravitational force, and the relevance of their emission in the bulk for the brane cosmology are analysed. Finally, constraints on parameters are derived from energy loss in astrophysical and particle physics processes.

Keywords: Extra Large Dimensions, p-branes, D-branes.

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Contents

1. Introduction

Quantum loop effects due to particles confined on a p-brane will induce a $p+1$-dimensional Einstein term, localized on the p-brane, in addition to the usual bulk terms in the effective action. Such a "lower-dimensional" term, in general absent at tree level in string theory and in effective supergravities, is certainly allowed by the symmetries of the theory not broken by the presence of the p-brane. If sizeable, it can result in major changes in the way the gravitational interaction is perceived on the brane.

A phenomenological approach to analyse these effects was followed in an example with a 3-brane embedded in a five-dimensional space-time bulk and the relevant part of the gravitational action parametrized as [1, 2, 3]

$$S = M^3 \int d^5x \sqrt{-g}\, R + M^3 r_c \int d^4x \sqrt{-\hat{g}}\, \hat{R}\,, \tag{1.1}$$

where $\hat{g}_{\alpha\beta}$, with $\alpha, \beta = 0, 1, 2, 3$, is the induced metric on the 3-brane. When the fifth dimension is assumed non-compact, the propagator of the graviton, as viewed by an observer on the brane, has the form $\sim M^{-3}(p + r_c p^2/2)^{-1}$, where p is the magnitude of its four-momentum along the 3-brane. Thus, at length scales $l \sim p^{-1} \gg r_c$ the gravitational potential behaves as in five dimensions, while in the opposite limit $l \sim p^{-1} \ll r_c$ it is effectively four-dimensional.

As pointed out in [2], an additional difference of this setup from standard compactification is that here the analogues of Kaluza-Klein states are weakly coupled to the brane fields and experiment provides less stringent constraints. In particular, upon compactification of the fifth dimension on a circle of radius R with $R \ll r_c$, the gravitational interaction is four-dimensional at all scales.[1] This can be understood from the fact that gravity is four-dimensional at length scales smaller than r_c, while the compact circle makes space-time effectively four-dimensional at length scales larger than R as well. There is an infinite discrete spectrum of graviton modes, with couplings to the brane fields that are suppressed compared to the usual compactification scenario. This modifies the appropriate experimental limits coming from solar system motions and energy loss in stars and supernovae, and allows values as exotic as $R \simeq 10^{-4} r_c \simeq 10^{16}$ m [2].

These results are valid for infinitely thin branes. The case of thick branes was studied in [5]. With ϵ being the brane thickness and M the five-dimensional Planck mass, it was shown (a) that at large distances the above picture remains the same, while (b) at distances smaller than $\sqrt{\epsilon/M}$ the Equivalence Principle is generically violated. To preserve it, one should fine tune the profiles of all particles on the brane. Similar conclusions were presented also in [6]. Moreover, in [5] the effects of R^2 terms, both in the bulk and on the brane, were analysed and shown to genericaly modify the behavior of gravity at all distances.

Gravity is expected to be induced on D-branes [7]. Higher-derivative gravitational terms on $N = 4$ D-branes have been calculated [8]. The induced one-loop Einstein term on the brane was recently calculated in the context of string theory [5]. In particular, it was shown that on a collection of D5-branes, with as much as $N = 2$ supersymmetry, an Einstein term is induced, with a coefficient that can be varied at will by varying the compactification moduli. Moreover, the transition to four-dimensional gravity can happen well below the string scale. This concrete realization has a much richer set of threshold scales than the naive five-dimensional model. The five-dimensional Planck scale appears as a "mirage" threshold: physics becomes ten-dimensional, and eventually ten-dimensional gravity becomes strong earlier. Also, it has been pointed out [9], that on D-branes in bosonic string theory the complete absence of supersymmetry is responsible for an induced Einstein term appearing already at tree level. The effects of that term are important at distances of the order of the string scale. Finally, the case of induced gravity on branes located in bulk space with more than one transverse dimensions has been analysed as well [3, 5, 10]. Attemps to use this in model building were also made [11].

So far, we have reviewed the physics of the *second* most important gravitational terms in the IR, namely the Einstein terms in the bulk and on the brane. We implicitly assumed that the cosmological constants of the bulk and of the brane, which are actually the leading terms in the IR, are both zero. The purpose of this work is to extend the above phenomenological analysis and study the effect of non-vanishing cosmological constants. We will mostly focus on a 3-brane world, embedded in a five-dimensional bulk and we will fine tune (à la RS [12]) the two cosmological constants so that the brane is flat.[2] Phrased in an equivalent way, we will be interested in the effects of the induced four-dimensional Einstein term on the RS scenario. The generic case of unrelated cosmological constants will be treated elsewhere.

[1] This may also happen without compactification, as argued in [4].

[2] For a recent discussion of exact solutions in the five-dimensional context with an induced Einstein term see [13].

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We have organized our paper in six sections, of which this introduction is the first. In section 2 we describe the effective action relevant to our discussion. We add the induced four-dimensional Einstein term to the action studied in [12], or equivalently, we include the two aforementioned cosmological terms in (1.1). We compute the graviton propagator as well as the gravitational potential due to a point source on the 3-brane and compare the results with the RS picture. Section 3 is devoted to the analysis of the spectrum of gravitational fluctuations around the brane background. The couplings of the corresponding eigenmodes to the brane matter are obtained here and their relative importance to the gravitational potential is analysed. The effects of a non-vanishing energy-momentum tensor on the brane and the main features of the corresponding cosmology are presented in section 4. The phenomenon of brane energy loss through the decay of ordinary matter to Kaluza-Klein gravitons into the bulk, together with its implications on astrophysical processes and on the cosmological evolution, are studied in section 5. Finally, section 6 contains our conclusions and a discussion of the potential relevance of our results.

2. Induced gravity in the Randall-Sundrum setup

We are interested in the effects of an induced four-dimensional Einstein term on the Randall-Sundrum (RS) brane configuration [12]. We consider a 3-brane in a five-dimensional bulk with a negative cosmological constant. Quantum loops of the particles confined on the brane will induce a four-dimensional Einstein term on the brane, which, as was pointed out in [1, 2], can have important effects on the effective gravitational interaction on it.

The relevant part of the low energy effective action is parametrized as

$$S = \int d^5x \, \sqrt{-g} \left(-\Lambda + M^3 R\right) + \int d^4x \sqrt{-\hat{g}} \left(-V_b + M^3 r_c \hat{R}\right) , \tag{2.1}$$

where $\hat{g}_{\alpha\beta}$, with $\alpha, \beta = 0, 1, 2, 3$, is the induced metric on the 3-brane. The fifth dimension is assumed to be non-compact. This corresponds to the limit that the negative-tension brane is taken to infinity [12]. We identify (x, z) with $(x, -z)$, where $z \equiv x_4$. However, following the conventions of [12] we extend the bulk integration over the entire interval $(-\infty, \infty)$. The quantity V_b includes the brane tension as well as quantum contributions to the four-dimensional cosmological constant. The strength of the induced Einstein term is parametrized in terms of the fundamental mass scale M and the length scale r_c [1].

Of the four a priori parameters M, Λ, V_b and r_c of (2.1), one (M) sets the scale, while for $V_b^2 = -12\Lambda M^3$ the corresponding equations of motion admit the standard RS solution

$$ds^2 = e^{2A(z)} \eta_{\alpha\beta} dx^\alpha dx^\beta + dz^2 \, , \tag{2.2}$$

with $A(z) = -k|z|$ and $k = -\Lambda/V_b$. We choose $1/k$ and r_c, both with dimensions of length, to be our two remaining parameters. We are interested in the graviton propagator in this background, corresponding to a perturbation in the metric for which $\eta_{\alpha\beta}$ is replaced by $\eta_{\alpha\beta} + h_{\alpha\beta}$ in the above expression. For simplicity we will ignore the tensor structure of the graviton field and solve instead for the massless scalar propagator $G(x, z)$, which is

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expected to capture the essential features of the gravitational correlations and satisfies

$$M^3\left[\frac{1}{\sqrt{-g}}\,\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\right)+\delta(z)\frac{r_c}{\sqrt{-\hat{g}}}\partial_\alpha\left(\sqrt{-\hat{g}}\,\hat{g}^{\alpha\beta}\partial_\beta\right)\right]G(x,z)=\delta^{(4)}(x)\delta(z)\,. \qquad (2.3)$$

In the background (2.2) in particular, it obeys

$$M^3\left(e^{-2A}\nabla_4^2-\partial_z^2-4A'\partial_z+r_c\delta(z)\nabla_4^2\right)G(x,z)=\delta^{(4)}(x)\delta(z)\,, \qquad (2.4)$$

where ∇_4^2 is the flat laplacian in four dimensions and the prime denotes differentiation with respect to z. Going to momentum space for the four-dimensional part we obtain the equivalent equation (we work in euclidean space with $p^0=-ip^5$)

$$M^3\left(e^{-2A}p^2-\partial_z^2-4A'\partial_z+r_c\delta(z)p^2\right)G(p,z)=\delta(z)\,, \qquad (2.5)$$

where $p^2=p_5^2+p_1^2+p_2^2+p_3^2$.

The symmetry of the problem implies that the propagator is a symmetric function of z, i.e. $G(p,z)=G(p,-z)$. The solution for $z>0$ is given by

$$G(p,z)=B\,w^2\,K_2\left(\frac{wp}{k}\right) \qquad (2.6)$$

with $w\equiv e^{kz}$ and K_2 the modified Bessel function. The multiplicative constant B is fixed by the requirement on the solution to satisfy the discontinuity condition

$$\left.\frac{\partial G(p,z)}{\partial z}\right|_{z=0+}-\left.\frac{\partial G(p,z)}{\partial z}\right|_{z=0-}=r_cp^2G(p,0)-\frac{1}{M^3} \qquad (2.7)$$

obtained from (2.5) by integrating both sides in the interval $(-,+)$ and taking the limit $\to 0$. We obtain

$$B^{-1}=M^3\left((-4k+r_cp^2)K_2\left(\frac{p}{k}\right)-2p\,K_2'\left(\frac{p}{k}\right)\right). \qquad (2.8)$$

One may use the identity $K_2'(z)+2K_2(z)/z=-K_1(z)$ to rewrite

$$B^{-1}=M^3p\left(2K_1\left(\frac{p}{k}\right)+r_cpK_2\left(\frac{p}{k}\right)\right). \qquad (2.9)$$

As we are interested in correlations on the brane we need

$$G(p,z=0)=\frac{1}{M^3p}\,\frac{K_2(p/k)}{2K_1(p/k)+r_cpK_2(p/k)}\,. \qquad (2.10)$$

We first reproduce the results of the RS case $r_c=0$. With $G(p,z)$ depending only on the magnitude p of the momentum, the corresponding potential due to the unit mass at $(x=0,z=0)$ as viewed by an observer on the 3-brane, at a distance r from the source, is

$$V(r)=\frac{1}{2\pi^2r}\int_0^\infty dp\,p\sin pr\,G(p,z=0)\,. \qquad (2.11)$$

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For the case at hand this leads to

$$V(r) = \frac{1}{4\pi^2}\frac{k}{M^3 r}\int_0^\infty d\tilde{p}\frac{K_2(\tilde{p})}{K_1(\tilde{p})}\sin(\tilde{p}kr) = \frac{1}{4\pi}\frac{k}{M^3}\frac{1}{r} + \delta V(r)\,, \tag{2.12}$$

where

$$\delta V(r) = \frac{1}{4\pi^2}\frac{k}{M^3 r}\int_0^\infty d\tilde{p}\frac{K_0(\tilde{p})}{K_1(\tilde{p})}\sin(\tilde{p}kr)\,. \tag{2.13}$$

In order to obtain the second equality above we used the identity $K_2(z) = 2K_1(z)/z + K_0(z)$.

For $\tilde{p} \to \infty$ the ratio $K_0(\tilde{p})/K_1(\tilde{p}) \to 1$ and the integral in (2.13) reduces to the ill-defined integral of $\sin\tilde{p}$ over the positive real axis. In order to evaluate (2.13) we multiply the integrand by $e^{-q\tilde{p}}$, perform the integration and then take the limit $q \to 0$.[3]

For $kr \gg 1$ the strongly oscillatory behavior of $\sin(\tilde{p}kr)$ results in a negligible contribution to the integral from large $\tilde{p}$. This means that we can employ the expansion of the Bessel functions for small $\tilde{p}$: $K_0(\tilde{p})/K_1(\tilde{p}) = -\tilde{p}\log\tilde{p}$. We obtain [12]

$$\delta V(r) \simeq \frac{1}{8\pi}\frac{1}{M^3 k}\frac{1}{r^3}\,. \tag{2.14}$$

Thus we reproduce the leading and subleading behavior of the potential at long distances in the RS scenario. For $kr \ll 1$ the main contribution to the integral comes from large $\tilde{p}$, for which $K_0(\tilde{p})/K_1(\tilde{p}) = 1$. We find

$$\delta V(r) \simeq \frac{1}{4\pi^2}\frac{1}{M^3}\frac{1}{r^2}\,. \tag{2.15}$$

Thus, at short distances gravity is *five-dimensional* [14]. Moreover we see that the behavior of the potential at small distances is different from the one implied by the subleading term for large kr.

We next study the general case $r_c \neq 0$. The Fourier transform is difficult to perform explicitly, but we can deduce the behavior of the potential by studying the behavior of the propagator as a function of p. We find that

$$\text{for } p \ll k\,, \qquad G(p, z=0) \simeq \frac{1}{M^3\,(r_c + 1/k)\,p^2}\,, \tag{2.16}$$

$$\text{for } p \gg k\,, \qquad G(p, z=0) \simeq \frac{1}{M^3\,(r_c p^2 + 2p)}\,. \tag{2.17}$$

Clearly, for $r_c \neq 0$ significant modifications of the gravitational potential are possible.

We will distinguish two separate cases:

(*a*) $kr_c \gg 1$: both for $p \ll k$ and $p \gg k$ we have $G^{-1} \simeq M^3 r_c p^2$. Thus we expect four-dimensional behavior $\sim 1/r$ for the potential at all distances on the brane, with an effective Planck constant $M_{Pl}^2 \simeq M^3 r_c$.

[3]The integral depends on the regulator. The result remains the same if instead of $e^{-q\tilde{p}}$ one multiplies by $e^{-\alpha\tilde{p}^2}$ and then takes the limit $\alpha \to 0$, while a simple ultraviolet cut-off L on the $\tilde{p}$ leads to an ambiguous answer. A correct regulator should reproduce the well known answer $V(r) \sim 1/r^2$ for the case $G(p, 0) \sim 1/p$, corresponding to five-dimensional behavior. The one we use satisfies this criterion. Equivalently, one could just use $\int_{-\infty}^{\infty} dx e^{i\alpha x} = 2\pi\delta(\alpha)$ to define the umbiguous integrals mentioned above.

The leading corrections to $V(r)$ can also be evaluated by employing the full propagator (2.10). We find

$$\begin{aligned} \text{for } r \gg 1/k\,, \qquad \delta V(r) &= \frac{1}{8\pi}\frac{1}{M^3 k}\frac{1}{(r_c k+1)^2}\frac{1}{r^3} \\ &\simeq \frac{1}{8\pi}\frac{1}{M_{Pl}^2}\frac{1}{r_c k^3}\frac{1}{r^3} \simeq \frac{1}{8\pi}\frac{1}{M_{Pl}^4}\left(\frac{M}{k}\right)^3\frac{1}{r^3}\,, \qquad (2.18) \\ \text{for } r \ll 1/k\,, \qquad \delta V(r) &= \frac{1}{\pi^2}\frac{1}{M^3 r_c^2}\log(kr) \\ &\simeq \frac{1}{\pi^2}\frac{1}{M_{\rm Pl}^2 r_c}\log(kr) \simeq \frac{1}{\pi^2}\frac{M^3}{M_{\rm Pl}^4}\log(kr)\,. \qquad (2.19) \end{aligned}$$

(*b*) $kr_c \ll 1$: for $p \ll k$, $G^{-1} \simeq M^3p^2/k$. We expect that at large distances $r \gg 1/k$ the potential displays four-dimensional behavior with $\tilde{M}_{\rm Pl}^2 \simeq M^3/k$, as in the standard RS scenario. For $k \ll p \ll 1/r_c$, we have $G^{-1} \simeq 2M^3p$. Thus, for distances $r_c \ll r \ll 1/k$ we expect five-dimensional behavior $\sim 1/r^2$ for the potential. This is in agreement with the direct evaluation of the potential for $r_c = 0$. Finally, for $p \gg 1/r_c$, $G^{-1} \simeq M^3r_cp^2$. At short distances $r \ll r_c$ the behavior becomes again four-dimensional $\sim 1/r$, with $M_{Pl}^2 \simeq M^3 r_c$.

To summarize, the four-dimensional Einstein term induced quantum mechanically on the 3-brane affects considerably the gravitational interactions on the brane. Specifically, the gravitational potential on the brane exhibits the four-dimensional behavior $V(r) \sim 1/r$, except in the intermediate region $r_c \ll r \ll 1/k$, in which it is effectively five-dimensional, given by $V(r) \sim 1/r^2$. Furthermore, for $kr_c \ll 1$ the strength of the gravitational interaction, i.e. the value of the effective M_{Pl}, depends on the distance between the interacting masses. It is stronger for short distances, the ratio of its value for $r \ll r_c$ to the one for large $r \gg 1/k$ being equal to kr_c.

3. Kaluza-Klein modes and their effects

The physical picture of the previous section can also be confirmed by considering the massive gravitons, i.e. the Kaluza-Klein modes of the effective compactification induced by the warped geometry. We ignore as before the tensor structure of the metric and denote by $\Phi(x^\alpha, z)$ the small fluctuation field around the background (2.2). Its equation of motion at the linearized level is

$$M^3\left[\frac{1}{\sqrt{-g}}\,\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\right) + \delta(z)\frac{r_c}{\sqrt{-\hat{g}}}\partial_\alpha\left(\sqrt{-\hat{g}}\,\hat{g}^{\alpha\beta}\partial_\beta\right)\right]\Phi(x^\alpha, z) = 0\,. \qquad (3.1)$$

As suggested by the symmetries of the background, we look for solutions in the form $\Phi(x^\alpha, z) = \sum_n \phi_n(z)\sigma_n(x^\alpha)$, where the $\sigma_n(x^\alpha)$ satisfy the four-dimensional Klein-Gordon equation $(\partial^\alpha\partial_\alpha + m_n^2)\sigma_n = 0$. Using this in (3.1), one is led to the field equation

$$\left(\partial_z^2 + e^{-2A}m_n^2 + 4A'\partial_z + r_c\delta(z)m_n^2\right)\phi_n(z) = 0 \qquad (3.2)$$

for the mode function $\phi_n(z)$.

The zero mode, the solution corresponding to $m^2 = 0$, is not affected by the presence of the term proportional to r_c and consequently is identical to the one in reference [12]. The effective four-dimensional squared Planck constant is determined by taking the low-energy limit of (1.1). The term $\sim M^3 R$ gives $(M^3/k)\hat{R}$ in the low-energy effective action [12]. This contribution combined with the term $M^3 r_c \hat{R}$ results in a low-energy theory with an effective squared Planck constant $M^3(r_c + 1/k)$, in agreement with eq. (2.16).

The KK modes, analogous to those of [12], are $\psi_n = \exp(3A/2)\,\phi_n$. For $A(z) = -k|z|$ eq. (3.2) gives (for simplicity we omit the index n from m_n and ψ_n)

$$\psi(z) = N(\tilde{m}) w^{1/2} \left[Y_2(\tilde{m}w) + F(\tilde{m})\, J_2(\tilde{m}w) \right], \tag{3.3}$$

with $w \equiv \exp(k|z|)$, $\tilde{m} = m/k$. The constant $F(\tilde{m})$ is fixed by the discontinuity in $\partial_z \phi(0)$ due to the presence of the δ-function

$$F(\tilde{m}) = -\frac{2Y_1(\tilde{m}) + \tilde{r}_c \tilde{m} Y_2(\tilde{m})}{2J_1(\tilde{m}) + \tilde{r}_c \tilde{m} J_2(\tilde{m})}, \tag{3.4}$$

with $\tilde{r}_c = r_c k$, and Y_n, J_n the Bessel functions in standard notation.

For $w \to \infty$ the KK modes become approximately plane waves

$$\begin{aligned} \psi(w) &\simeq N(\tilde{m}) \sqrt{\frac{2}{\pi \tilde{m}}} \left[\sin\left(\tilde{m}w - \frac{5}{4}\pi \right) + F(\tilde{m}) \cos\left(\tilde{m}w - \frac{5}{4}\pi \right) \right] \\ &\simeq N(\tilde{m}) \sqrt{\frac{2\left(1 + F^2(\tilde{m})\right)}{\pi \tilde{m}}} \sin\left(\tilde{m}w - \frac{5}{4}\pi + \beta(\tilde{m}) \right), \end{aligned} \tag{3.5}$$

with $\beta = \arctan F$. As a result, for a non-compact fifth dimension the KK modes have a continuous spectrum and their normalization is approximately that of plane waves

$$N(\tilde{m}) \sim \sqrt{\frac{\tilde{m}}{1 + F^2(\tilde{m})}}, \tag{3.6}$$

where we have neglected factors of order 1. The strength of the interaction of the KK graviton modes with the other fields on the brane is determined by the square of their wavefunction at the position $z = 0$ of the brane. We find

$$\psi(z=0) \sim \sqrt{\frac{\tilde{m}}{1 + F^2(\tilde{m})}} \left[Y_2(\tilde{m}) + F(\tilde{m})\, J_2(\tilde{m}) \right]. \tag{3.7}$$

A careful examination of the low energy effective action reveals the presence of an additional suppression factor. The second term in the action (1.1) results in a non-canonical kinetic term for the fields $\sigma_n(x^\alpha)$. In order to render this term canonical we must absorb a factor $(1 + \tilde{r}_c |\psi(0)|^2)^{1/2}$ into the redefinition of the fields. The proof is completely analogous to the derivation of eq. (7.16) of [2]. This results in a suppression of all interactions with external sources by the same factor. However, this correction is negligible in all the cases we study below.

After the KK kinetic terms have been properly normalized, the coupling of the KK modes to matter on the brane is given by $\sqrt{k/M^3}$. This coupling is squared in the calculation of quantities such as KK mode production rates etc. It is also accompanied by the integration over all KK states with a plane-wave measure dm/k. As a result, the combination dm/M^3 appears in all the estimates of KK mode production in the following.

For $\tilde{m} \ll 1$ eq. (3.7) gives

$$\psi(z=0) \sim \frac{\sqrt{\tilde{m}}}{1+\tilde{r}_c}. \tag{3.8}$$

We thus recover the suppression $\sim \sqrt{m/k}$ of the standard Randall-Sundrum model, which is further enhanced for large kr_c.

For $\tilde{m} \gg 1$, on the other hand, we find

$$\psi(z=0) \sim \left[1+\left(\frac{\tilde{r}_c\tilde{m}}{2}\right)^2\right]^{-1/2}. \tag{3.9}$$

For $m \gg 1/r_c$ there is a significant suppression factor $\sim 1/(mr_c)$, while for $m \ll 1/r_c$ the wavefunction on the brane is unsupressed of order 1.

As we will show next, these results are consistent with the findings of the previous section and, in addition, crucial to clarify the origin of the behavior of the effective gravitational interaction on the brane. We will again separate the two different cases: (a) $kr_c \gg 1$, where gravity was found to be four-dimensional at all distances; (b) $kr_c \ll 1$, where gravity again behaves as in four dimensions, except in the intermediate range $(k, 1/r_c)$ where it is five-dimensional.

(a) $kr_c \gg 1$: for $m \lesssim k$ the wavefunction of the KK modes on the brane is $\sim \sqrt{m/(r_c^2k^3)}$, while for $m \gtrsim k$ it is $\sim 1/(mr_c)$. Thus, the gravitational potential is dominated by the exchange of the zero mode and falls off $\sim 1/r$ for all r. The effective squared Planck constant is $M^3(r_c+1/k) \simeq M^3 r_c$.

(b) $kr_c \ll 1$: the wavefunction of modes with $m \lesssim k$ is $\sim \sqrt{m/k}$, that of modes with $k \lesssim m \lesssim 1/r_c$ is of order 1, while that of modes with $m \gtrsim 1/r_c$ is $\sim 1/(mr_c)$.

i) For $r \gtrsim 1/k$ the corrections to the four-dimensional potential are dominated by modes with $m \lesssim k$ because the contribution of modes with $m \gtrsim k$ is exponentially suppressed. The contribution of massive modes is negligible relative to that of the zero mode. Thus we expect a fall-off $\sim 1/r$ with a squared Planck constant $M^3(r_c+1/k) \simeq M^3/k$.

ii) For $r_c \lesssim r \lesssim 1/k$ only the modes with $m \lesssim 1/r_c$ contribute significantly. Those with $k \lesssim m \lesssim 1/r_c$ generate a term in the potential

$$\delta V(r) \sim \frac{1}{M^3}\int_k^{1/r_c} dm \frac{e^{-mr}}{r}\psi^2(0) \sim \frac{1}{M^3}\int_k^{1/r_c} dm \frac{e^{-mr}}{r} \simeq \frac{1}{M^3 r^2}. \tag{3.10}$$

This contribution is much larger than those from the modes with $m \lesssim k$ and the zero mode. For example the modes with $m \lesssim k$ give

$$\delta V(r) \sim \frac{1}{M^3}\int_0^{k} dm \frac{e^{-mr}}{r}\frac{m}{k} \simeq \frac{k}{M^3 r}. \tag{3.11}$$

Thus, for distances $r_c \lesssim r \lesssim 1/k$ we expect five-dimensional behavior $\sim 1/r^2$ for the potential.

iii) Finally, at short distances $r \lesssim r_c$ the modes with $m \gtrsim 1/r_c$ give a contribution

$$\delta V_1(r) \sim \frac{1}{M^3} \int_{1/r_c}^{\infty} dm \frac{e^{-mr}}{r} \frac{1}{m^2 r_c^2} \simeq \frac{1}{M^3 r_c r} . \tag{3.12}$$

Those with $1/r_c \gtrsim m \gtrsim k$ give

$$\delta V_2(r) \sim \frac{1}{M^3} \int_{k}^{1/r_c} dm \frac{e^{-mr}}{r} \simeq \frac{1}{M^3 r_c r} . \tag{3.13}$$

These dominate over the contribution of the zero mode, as well as of the modes with $m \lesssim k$. Thus, the potential obtains again the four-dimensional form $\sim 1/r$, with a squared Planck constant $M^3 r_c$. It is remarkable that this behavior is not due to the zero mode, as on might have guessed, but instead it is attributed to the exchange of massive modes with masses $m \gtrsim k$. Similar behavior was also observed in [15, 16].

4. Cosmology

The Friedmann equations for the RS cosmology, as modified by the induced Einstein term, were derived by Deffayet [17] following the methods of [18]. Here, we will quote the result and analyse it in a context not studied in the existing literature [19]. The presence on the brane of matter/radiation and of a non-vanishing cosmological constant, with total energy density ρ_b and pressure p_b, results in a four-dimensional Friedmann-Robertson-Walker solution for the metric of the form

$$ds^2 = e^{2A(z)} \left[-dt^2 + a^2(t) \left(\frac{dr^2}{1 - \lambda r^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right) \right] + dz^2 . \tag{4.1}$$

The scale factor $a(t)$, the Hubble constant $H = \dot{a}/a$ as viewed by an observer on the brane, and the spatial curvature $\lambda = \pm 1, 0$ are related by the Friedmann equation

$$\frac{r_c^2}{2} \left(H^2 + \frac{\lambda}{a^2} \right) = 1 + \frac{r_c}{12 M^3} \rho_b + \epsilon \sqrt{1 + \frac{r_c}{6 M^3} \rho_b - \frac{r_c^2}{12 M^3} \Lambda + \frac{r_c^2 C}{a^4}} . \tag{4.2}$$

C is an integration constant [18]. If non-zero, it generates a mirage radiation density on the brane [20]. In this work we set $C = 0$.

The parameter $\epsilon = \pm 1$ defines two possible branches in the solution. Its value is determined by the sign of dA/dz at $z = 0$ [17], which must be negative for the graviton zero mode to be localized on the brane. Thus, cosmology on a brane with four-dimensional gravity at large distances requires $\epsilon = -1$. As this is the scenario of interest to us, we set $\epsilon = -1$ in the following. The case $\epsilon = 1$ was discussed in [19] where it was pointed out that it generates a late time cosmological constant.

For simplicity we will assume, for the time being, that there in no significant flow of energy out of the brane through the decay of brane matter into KK modes of the graviton.[4] Under this assumption, the energy density ρ_b on the brane satisfies the conservation

[4]The validity of this assumption is not guaranteed for large energy densities when $k \ll 1/r_c$, as the KK modes with $k \lesssim m \lesssim 1/r_c$ are unsuppressed on the brane. We will come back to this point in the next section, where we will estimate the energy loss.

equation

$$\dot{\rho}_b = -3H(\rho_b + p_b)\,. \tag{4.3}$$

We are studying here the effects induced by the presence of brane matter and radiation on the RS "vacuum" background (2.2) with a given value of k, i.e. with the parameters Λ and V_b of the theory satisfying $V_b = -\Lambda/k = 12M^3k$. As a consistency check, it is straightforward to verify that (4.2) is satisfied by (2.2) for $\rho_b = V_b = -p_b$, $\epsilon = -1$, and $C = \lambda = 0$. Separating the energy density as $\rho_b = V_b + \rho$ and using the above values for Λ and V_b, (4.2) takes the form

$$\frac{r_c^2}{2}\left(H^2 + \frac{\lambda}{a^2}\right) = 1 + kr_c + \frac{r_c}{12M^3}\rho - \sqrt{(1+kr_c)^2 + \frac{r_c}{6M^3}\rho}\,. \tag{4.4}$$

This equation has a smooth limit as $r_c \to 0$ and gives the cosmological evolution of the RS universe

$$H^2 = \frac{k\rho}{6M^3} + \frac{1}{4}\left(\frac{\rho}{6M^3}\right)^2 - \frac{\lambda}{a^2}\,. \tag{4.5}$$

In analysing the physical content of (4.4) we will distinguish two cases:

(a) The case $kr_c \gg 1$. For the gravitational potential this corresponds to four-dimensional behavior on the brane at all scales with $M_{\rm Pl}^2 = M^3 r_c$. We define the normalized density $\tilde{\rho} \equiv r_c\rho/M^3$. We have

i) for $\tilde{\rho} \gg (kr_c)^2$ or $\tilde{\rho} \simeq (kr_c)^2$ or $\tilde{\rho} \ll (kr_c)^2$:

$$H^2 \simeq \frac{\rho}{6M_{\rm Pl}^2} - \frac{\lambda}{a^2}\,, \tag{4.6}$$

Thus, we obtain at all times the standard Friedmann equation.

(b) The case $kr_c \ll 1$. The gravitational potential on the brane is four-dimensional at energies $E \ll k$ with $\tilde{M}_{\rm Pl}^2 = M^3/k$ (RS regime), and for $E \gg 1/r_c$ with $M_{\rm Pl}^2 = M^3 r_c$ (Induced Gravity (IG) regime). At intermediate energies $k \ll E \ll 1/r_c$ gravity is five-dimensional ($5d$ regime). We obtain

i) $\tilde{\rho} \gg 1$ (or $\rho/M^3 \gg 1/r_c$):

$$H^2 \simeq \frac{\rho}{6M_{\rm Pl}^2} - \frac{\lambda}{a^2}\,, \tag{4.7}$$

corresponding to the IG regime.

ii) $\tilde{\rho} \ll 1$:

$$H^2 \simeq \frac{\rho}{6\tilde{M}_{\rm Pl}^2} + \frac{1}{4}\left(\frac{\rho}{6\tilde{M}_{\rm Pl}^2 k}\right)^2 - \frac{\lambda}{a^2}\,. \tag{4.8}$$

We recover the cosmology of the RS universe. The range $\rho/M^3 \ll k$ corresponds to the RS regime

$$H^2 \simeq \frac{\rho}{6\tilde{M}_{\rm Pl}^2} - \frac{\lambda}{a^2}\,, \tag{4.9}$$

while for the range $k \ll \rho/M^3 \ll 1/r_c$ the term $\sim \rho^2$ in eq. (4.8) is important ($5d$ regime).

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Thus, we can confirm that the rough cosmological evolution is analogous to the static behavior of gravity on the brane. The various regimes can be discussed in terms of an energy scale set by $1/r$ or ρ/M^3, for the static potential or cosmology respectively. However, we have neglected a potentially important factor: the energy that is radiated off the brane into the bulk in the form of KK gravitons. We will study this in more detail in the next section.

5. Emission of Kaluza-Klein modes and experimental constraints

Matter on the brane can loose energy by emitting KK gravitons. Here we will estimate the efficiency of such processes in various contexts.

We will again separate the two different cases:

(a) $kr_c \gg 1$. This case is not very constrained by experiment. All KK modes are significantly suppressed and do not affect standard processes. For example, the rate of emission of KK modes from a star can be estimated as

$$\Gamma(T) \propto \frac{1}{M^3}\int_0^T dm\,\psi(0)^2 \sim \frac{1}{M^3}\int_0^T dm\frac{m}{k}\frac{1}{r_c^2k^2} \sim \frac{1}{M_{\rm Pl}^2}\frac{T^2}{k^2}\frac{1}{r_ck} \tag{5.1}$$

for $T < k$. This is much smaller than the rate of production of zero-mode gravitons $\Gamma_0(T) \propto 1/M_{\rm Pl}^2$, and is negligible. For $T > k$ the largest contribution to the rate is

$$\Gamma(T) \propto \frac{1}{M^3}\int_k^T dm\frac{1}{m^2r_c^2} \sim \frac{1}{M_{\rm Pl}^2}\frac{1}{r_ck}\,, \tag{5.2}$$

and is negligible again. Thus there are no severe constraints on the parameters apart from the requirement to reproduce the value of the Planck constant $M_{\rm Pl}^2 = M^3r_c$.

The most interesting property of the scenario concerns the corrections to the gravitational potential. In particular, the form of the corrections changes at a characteristic distance $\sim 1/k$, according to eqs. (2.18),(2.19). Since k is largely unconstrained by other considerations it is essentially a free parameter. The logarithmic variation at small distances is probably too slow to be detected. The $1/r^3$ correction to Newton's law at large distances is constrained by experiment [21, 22]. If the potential is parametrized as

$$V(r) = CN_1N_2\frac{(10^{-15}\ \mathrm{m})^2}{r^3}\,, \tag{5.3}$$

with N_1, N_2 the number of nucleons of the two interacting objects, the strongest bound is $C \leq 7\times10^{-7}$. Comparing with eq. (2.18) we obtain $k/M \gtrsim 10^{-20}$.

Since the four-dimensional Planck scale is related to r_c for $r_c \gg 1/k$ there are enough free parameters to make the values of Λ, V_b natural. Choosing $k = M = M_{\rm SUSY}$ indeed accomplishes this. r_c is determined from $M_{Pl}^2 = M^3r_c$. However, it should be remembered that, in a given theory, r_c is determined by the loop corrections and is not a free parameter.

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The cosmological evolution in this scenario is standard for all densities. However, there is a small amount of energy loss to the bulk. The change in energy density per unit time is equal to the rate of energy loss to KK gravitons per unit time and volume. For a process $a + b \to c + KK$ it is given by

$$\left(\frac{d\rho}{dt}\right)_{\text{lost}} = - \langle n_a \, n_b \, \sigma_{a+b\to c+KK} \, v \, E_{KK} \rangle \, , \tag{5.4}$$

where the brackets indicate thermal averaging. For a radiation-dominated brane we can take approximately $n_a, n_b \sim T^3$, $E_{KK} \sim T$, and estimate

$$\langle \sigma_{a+b\to c+KK} \, v \rangle \sim \frac{1}{M^3} \int_k^T dm \frac{1}{m^2 r_c^2} \sim \frac{1}{M_{\rm Pl}^2} \frac{1}{r_c k} \, , \tag{5.5}$$

in agreement with eq. (5.2). This leads to

$$\left(\frac{d\rho}{dt}\right)_{\text{lost}} \sim -\frac{T^7}{M_{\rm Pl}^2} \frac{1}{r_c k} \sim -\frac{\rho^{7/4}}{M_{\rm Pl}^2} \frac{1}{r_c k} \, . \tag{5.6}$$

We conclude that the energy loss is negligible because $(d\rho/dt)_{\text{expansion}}/(d\rho/dt)_{\text{lost}} \sim (M_{\rm Pl}/\rho^{1/4}) r_c k \gg 1$. Thus, we obtain a standard Friedmann cosmological expansion with essentially no energy loss.

(b) For $k r_c \ll 1$ the deviations from the standard RS physics appear at energy scales much larger than k. For the gravitational potential, we expect a transition from the four-dimensional form $\sim 1/r$ to the five-dimensional one $\sim 1/r^2$ at distances $r \lesssim 1/k$. The experimental constraints require $k \gtrsim (10 \; \mu\text{m})^{-1} \simeq 10^{-11}$ GeV, while the value of M is fixed by the relation $\tilde{M}_{\rm Pl}^2 = M^3/k$ to be $M \gtrsim 10^9$ GeV.

The emission of KK modes with masses $1/r_c \gtrsim m \gtrsim k$ is unsuppressed on the brane. Their contribution to various processes, such as star cooling or high-energy experiments, is analogous to those in standard torroidal compactifications, [22, 23] with one extra dimension and a compactification radius $\sim 1/k$. The strongest constraints arise from star cooling through the emission of KK modes. For $1/r_c > T > k$ the largest contribution is

$$\Gamma(T) \propto \frac{1}{M^3} \int_0^T dm \, \psi(0)^2 \sim \frac{1}{M^3} \int_k^T dm \sim \frac{1}{\tilde{M}_{\rm Pl}^2} \frac{T}{k} \, . \tag{5.7}$$

For the case of a supernova with $T \sim 30\,\text{MeV}$, we can require that this rate be smaller than the equivalent axion production rate $\Gamma_a \propto 1/f_a^2$, for which the constraint is $f_a \gtrsim 10^9$ GeV [22]. This leads to $k \gtrsim 10^{-21}$ GeV, a much weaker bound than the one imposed by deviations from newtonian gravity. In a more careful treatment one should take into account the details of the various processes of KK graviton production, such as nucleon-nucleon Brehmstrahlung, gravi-Compton scattering etc. However, the basic conclusion that the resulting bound is much weaker than the one imposed by deviations from newtonian gravity is expected to be unaffected. The situation with $T > 1/r_c$ leads to more relaxed constraints because of the suppression of KK modes with $m \gtrsim 1/r_c$ on the brane.

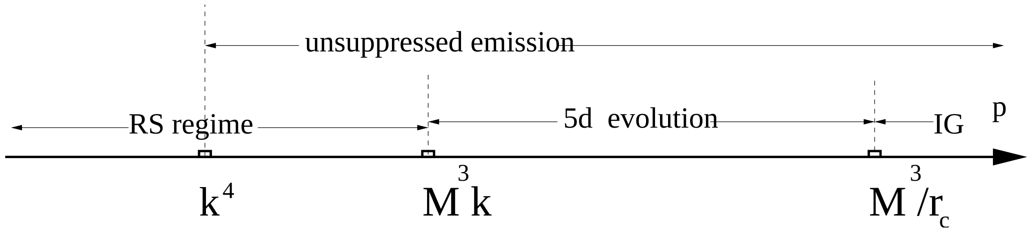

Figure 1: Different regions in density, during the evolution of the universe.

For $k \gtrsim 10^{-11}\,\mathrm{GeV}$, $M \gtrsim 10^9\,\mathrm{GeV}$ we can evaluate the vacuum energy in the bulk $|\Lambda| \gtrsim (10\,\mathrm{GeV})^5$ and the brane $V_b \gtrsim (10\,\mathrm{TeV})^4$. The length r_c is not constrained and determines the weakness of gravity at short distances $\tilde{M}_{\mathrm{Pl}}/M_{\mathrm{Pl}} = \sqrt{kr_c} \ll 1$.

Note that the lowest values of V_b are compatible with the possibility that it arises through supersymmetry breaking on the brane with M_{SUSY} of the order of 10 TeV. Moreover, it is expected that it will introduce a bulk supersymmetry breaking scale of the order of $M^2_{\mathrm{SUSY}}/M \sim 0.1\,\mathrm{GeV}$. This scale is about 100 times smaller than the one needed to equilibrate and flatten the brane.

The cosmology of this scenario has several novel features, as was indicated in the previous section. For densities $\rho \lesssim M^3 k$ one expects the standard cosmological evolution with $H^2 \sim \rho/\tilde{M}^2_{\mathrm{Pl}}$ (RS regime). For k, M near the lower bound set by observations $k \sim 10^{-11}$, $M \sim 10^9\,\mathrm{GeV}$, this regime extends up to densities $\rho \sim (10\,\mathrm{TeV})^4$. However, for $M^3 k \lesssim \rho \lesssim M^3/r_c$ the Hubble parameter behaves $H^2 \sim \rho^2/M^6$ ($5d$ regime), while for $\rho \gtrsim M^3/r_c$ we have $H^2 \sim \rho/M^2_{\mathrm{Pl}}$ (IG regime).

As shown in the previous section, unsuppressed emission of single KK gravitons can take place for the mass range $1/r_c > m > k$. For a brane with energy density $\rho \gtrsim k^4$ it is possible to produce such unsuppressed KK gravitons that escape into the bulk. We concentrate on the case of a radiation-dominated brane-universe ($\rho \sim T^4$), which is the most relevant for the energy scales of interest. The scale k^4 is smaller than $M^3 k$ because we assume $k \ll M$ (otherwise the whole energy regime above k is strongly coupled). We also assume $Mr_c \gg 1$ (otherwise induced-gravity effects are masked by strong five-dimensional gravity).

The energy lost through emission of unsuppressed KK gravitons is given by eq. (5.4). We estimate

$$\langle \sigma_{a+b\to c+KK}\, v \rangle \sim \frac{1}{M^3}\int_k^{\min(T,1/r_c)} dm = \frac{\min(T,1/r_c)}{M^3}\,. \tag{5.8}$$

For a given temperature T, the energy loss is maximized if $1/r_c > T$. We concentrate on this case in the following and find

$$\left(\frac{d\rho}{dt}\right)_{\mathrm{lost}} \sim -\frac{T^8}{M^3} \sim -\frac{\rho^2}{M^3}\,. \tag{5.9}$$

The change in energy density because of the expansion is

$$\left(\frac{d\rho}{dt}\right)_{\mathrm{exp}} = -4H\rho\,. \tag{5.10}$$

In the RS regime $(d\rho/dt)_{\rm exp}/(d\rho/dt)_{\rm lost} \sim (M^3k/\rho)^{1/2} \gg 1$. Thus the energy loss during this period is negligible compared to the dilution due to the expansion. In particular, for $M^3k \sim (10\,\mathrm{TeV})^4$, the energy loss during nucleosynthesis is smaller by a factor 10^{14} than the rate of decrease of the energy density because of the expansion. Thus, during this period the standard cosmological evolution is not affected by energy loss. A differed way of saying this is that graviton emission is frozen out during the RS period.

The energy loss is substantial during the $5d$ regime, when $(d\rho/dt)_{\rm exp} \sim -\rho^2/M^3$. Both $(d\rho/dt)_{\rm exp}$ and $(d\rho/dt)_{\rm lost}$ are of the same order of magnitude and both lead to a decrease of the energy density for an expanding universe. Parametrizing

$$\left(\frac{d\rho}{dt}\right)_{\rm lost} = -\sigma\frac{\rho^2}{M^3} \tag{5.11}$$

we find

$$a(t) \sim t^{\frac{1}{4(3\sigma+1)}}\,, \qquad \rho \sim \frac{1}{a^{4(1+3\sigma)}}\,, \qquad H = \frac{\rho}{12M^3} \tag{5.12}$$

during the $5d$ period. Therefore, the expansion slows down and the radiation behaves effectively as a fluid satisfying an equation of state $p = w_{\rm eff}\ \rho$ where $w_{\rm eff} = \frac{1}{3} + 4\sigma$.

A more interesting effect can appear if the universe is contracting during the $5d$ regime: $H = -\rho/(12M^3)$. In this case

$$\rho \sim \frac{1}{a^{4(1-3\sigma)}}\,. \tag{5.13}$$

If $\sigma < 1/3$ the universe will reach a big-crunch singularity $a = 0$ in a finite time. On the other hand if the radiation loss is important, $\sigma > 1/3$, the contraction of the universe will asymptotically slow down

$$a(t) \sim t^{-\frac{1}{4(3\sigma-1)}}\,, \tag{5.14}$$

while the universe is dumping all its energy in the bulk.

In our discussion we assumed that $1/r_c > T$, so that the energy loss is maximized. (For this to hold during part of the $5d$ regime, we must have $r_c^{-4} > M^3k$.) In the opposite case $1/r_c > T$, the energy loss is smaller by a factor of r_cT. As a result, it remains negligible during the RS regime, while it can never prevent the collapse of a contracting universe during the $5d$ regime. Finally, it can be checked that the energy loss is smaller than the energy dilution through expansion during the IG regime if $1/r_c < M$.

6. Conclusions

We have analysed some of the physics of the RS 3-brane universe embedded in five dimensions in the presence of the four-dimensional Einstein term, induced by quantum loops of particles confined on the brane. The scenario is characterized by the two length scales $1/k$ and r_c, associated with the cosmological constants and the induced Einstein term respectively. As we argued, the presence of the latter may affect considerably the short-distance structure of gravity on the brane.

Specifically, we showed that the four-dimensional $\hat{R}$ term in (2.1) dominates the gravitational potential for all distances smaller than the characteristic scale r_c associated with it, and results in the usual $\sim 1/r$ four-dimensional behavior. For $r \gg r_c$ its role is subdominant compared to the five-dimensional Einstein term. Thus, it modifies the short distance behavior of the gravitational potential of the RS scenario from five- to four-dimensional, while it leaves its large distance four-dimensional form unaffected.

Our conclusions for the gravitational interaction on the brane, for the two generic cases that parallel the situation with toroidal compactification plus induced four-dimensional gravity [2], are summarized as follows:

(a) $kr_c \gg 1$. Gravity on the 3-brane is four-dimensional at all distances.

(b) $kr_c \ll 1$. Gravity on the 3-brane is four-dimensional at long and short distances, except for the intermediate range $(k, 1/r_c)$ where it exhibits five-dimensional behavior.

We have analysed the spectrum and couplings of the KK modes of the graviton. We find that in the regimes where induced gravity is dominant there is a further supression of their couplings. The couplings are not supressed only in the mass range that corresponds to gravity having a five-dimensional behavior.

We have further analysed the cosmological evolution equations in the standard branch in the presence of bulk and brane cosmological constants. We show that in case (a) the cosmological evolution at late times is very similar to the standard four-dimensional Friedmann evolution. In case (b) there is an intermediate period where the cosmological evolution is five-dimensional [18], namely $H^2 \sim \rho^2$, while for $\rho \ll M^3 k$ we recover the standard four-dimensional Friedmann equation $H \sim \rho$. There is also an earlier epoch during which the evolution is again four-dimensional but with a different Planck scale $M_{Pl}^2 = M^3 r_c$.

An interesting effect that has to be added to the standard evolution equations is the leakage of energy density from the brane to the bulk due to (massive) graviton emission. This has been estimated using the results of section 3. It turns out that, for all periods in which the Friedmann equation is effectively four-dimensional ($H^2 \sim \rho$), the leakage contribution to the time derivative of the brane energy density is negligible compared to the "dilution" term $-3H(\rho + p)$. Thus, normal four-dimensional intuition applies and no constraints from succesful late time events (nucleosynthesis, CMBR) are important. During the $5d$ period on the other hand, the leakage term is as important as the dilution term. This modifies the nature of the expansion, by slowing it down further. If the brane-universe is contracting during that period, and the coefficient of this term is larger than that of the dilution term, the universe contraction stops asymptotically.

With respect to the effect of phenomenological constraints on the parameters of the model, the situation can be summarized as follows:

Case (a) is not severely constrained by experiment except for the requirement to reproduce the Planck scale $M_{Pl}^2 = M^3 r_c$. Constraints on the subleading terms of the gravitational interaction impose $k/M \gtrsim 10^{-20}$. Moreover, there is enough free parameter space to make the values of bulk and brane cosmological constants natural. Choosing $k = M = M_{\rm SUSY}$ indeed accomplishes this. However, it should be remembered that in a given theory r_c is not a free parameter, but is determined by the loop corrections.

Case (b) is more tightly constrained. From experimental constraints, the energy scale must satisfy $k \gtrsim 10^{-11}$ GeV corresponding to distances shorter than 10 nm. Thus, at large distances the strength of gravity is determined by $\tilde{M}_{Pl} = \sqrt{M^3/k} \simeq 10^{19}$ GeV, implying $M \gtrsim 10^9$ GeV. We can evaluate the vacuum energy in the bulk $\Lambda \gtrsim (10^{2/5}\,\text{GeV})^5$ and the brane $V_b \gtrsim (10\,\text{TeV})^4$. The length scale r_c is not constrained and determines the strength of gravity at short distances $\tilde{M}_{Pl}/M_{Pl} = \sqrt{kr_c} \ll 1$. Note that gravity becomes stronger at distances beyond $1/r_c$. Moreover, the cosmological evolution is indistiguishable from the standard one except for very early times.

The lowest values of V_b are compatible with the possibility that it arises through supersymmetry breaking on the brane with M_{SUSY} of the order of 10 TeV. Moreover, it is expected that it will introduce a bulk supersymmetry breaking scale of the order of $M^2_{\text{SUSY}}/M \sim 0.1$ GeV. This scale is 100 times smaller than the one needed to equilibrate and flatten the brane.

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